Exercises on the chapters "Newton iteration for power series" & "Fast Evaluation and Interpolation"

To prepare for 8 October 2025

In what follows, $\mathbb{K}$ denotes an arbitrary field of characteristic zero.

Exercise 1. Assume that $F \in \mathbb{K}[[x]]$ with $F(0) = 1$.

(a) What is the complexity of computing $\sqrt{F}$, by using $\sqrt{F} = \exp(\frac{1}{2}\log F)$?

(b) Describe a Newton iteration that directly computes $\sqrt{F}$, without appealing to successive logarithm and exponential computations.

(c) Estimate the complexity of the algorithm in (b).

Exercise 2. Let f and g be two polynomials in $\mathbb{K}[x, y]$ of degrees at most d_x in x and at most d_y in y.

(a) Show that it is possible to compute the product $h = fg$ using

$$O(\mathsf{M}(d_x d_y))$$

arithmetic operations in $\mathbb{K}$.
Hint: Use the substitution $x \leftarrow y^{2d_y+1}$ to reduce the problem to the product of univariate polynomials.

(b) Improve this result by proposing an evaluation-interpolation scheme which allows the computation of h in

$$O(d_x\,\mathsf{M}(d_y) + d_y\,\mathsf{M}(d_x))$$

arithmetic operations in $\mathbb{K}$.