Exercise on the chapter
“Fast Evaluation and Interpolation”

To prepare for 2023-10-19

In what follows, $\mathbb{K}$ denotes an arbitrary field.

**Exercise 1.** Let $f$ and $g$ be two polynomials in $\mathbb{K}[x, y]$ of degrees at most $d_x$ in $x$ and at most $d_y$ in $y$.

(a) Show that it is possible to compute the product $h = fg$ using

$$O(M(d_xd_y))$$

arithmetic operations in $\mathbb{K}$.  

*Hint:* Use the substitution $x \leftarrow y^{2d_y+1}$ to reduce the problem to the product of univariate polynomials.

(b) Improve this result by proposing an evaluation-interpolation scheme which allows the computation of $h$ in

$$O(d_x M(d_y) + d_y M(d_x))$$

arithmetic operations in $\mathbb{K}$.