Exercise on the chapter "Fast Evaluation and Interpolation"

To prepare for 28 October 2024

In what follows, $\mathbb{K}$ denotes an arbitrary field.

Exercise 1. Let f and g be two polynomials in $\mathbb{K}[x, y]$ of degrees at most d_x in x and at most d_y in y.

(a) Show that it is possible to compute the product $h = fg$ using

$$O(\mathsf{M}(d_x d_y))$$

arithmetic operations in $\mathbb{K}$.
Hint: Use the substitution $x \leftarrow y^{2d_y+1}$ to reduce the problem to the product of univariate polynomials.

(b) Improve this result by proposing an evaluation-interpolation scheme which allows the computation of h in

$$O(d_x\, \mathsf{M}(d_y) + d_y\, \mathsf{M}(d_x))$$

arithmetic operations in $\mathbb{K}$.