

Computational Differential Algebra for the Enumeration of Regular Bipartite Graphs

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Scientific context: Over the past decades, computer algebra has made major advances in the development of algorithms for exact computations with differential and recurrence equations and systems. This includes:

- the algorithmic theory of non-commutative Gröbner bases for systems of functional equations,
- efficient algorithms for computing parameterized integrals and sums,
- tools for solving or simplifying differential and recurrence equations,
- and methods for deriving asymptotic expansions of their solutions.

Combinatorics has long been a fruitful playground for these techniques. Many combinatorial problems have been solved by computer-algebraists using their tools, and conversely, combinatorialists have often embraced computer-algebra methods to push their own research.

A prominent example of this synergy is the enumeration of constrained families of graphs. The standard approach in combinatorics is to introduce the generating function of a graph family, a formal power series in t where the coefficient of t^n gives the number of such graphs on n vertices. Computer algebra is then used to obtain information on the generating function. For a specific context, k -regular graphs are those in which every vertex has exactly k neighbors. Combining symmetric function theory with differential elimination, one can derive differential equations satisfied by the generating functions of k -regular graphs, for increasing values of k . The obtained equations make it then possible to compute counting sequences for large n . The difficulty of the problem grows quickly with k : only recently has a 20-year-old barrier been overcome, extending the record from $k = 4$ up to $k = 8$.

The same algebraic techniques should apply to many other combinatorial structures: variants of graphs, hypergraphs, maps, standard Young tableaux, lattice paths, etc. This internship will focus on bipartite graphs, whose vertices are colored black and white with no edge connecting two vertices of the same color. Beyond their theoretical interest, bipartite graphs model many natural problems: matchings (workers/jobs, doctors/hospitals), recommendation systems, scheduling (tasks/resources), and more.

Objectives of the internship: The internship aims to extend recent methods developed for uncolored graphs to the case of bipartite graphs. The objectives are:

- to generalize and implement the relevant algebraic algorithms,
- to compute, in a fully rigorous way, differential equations for k -regular bipartite graphs, for the largest possible values of k ,
- and to use these equations to extend known enumerative sequences, such as [A008327](#) and several related entries in the OEIS.

The work will start with a study of the existing literature, followed by the choice of the most suitable algorithmic variant to generalize, and finally its adaptation to the bipartite case.

Application requirements: Applicants should have a strong interest in both mathematics and algorithms, with an appetite for theory as well as computer experimentation. Background in at least one of the following areas will be valuable: Gröbner bases, effective polynomial elimination, or symmetric function theory. Some programming experience in a high-level algebraic language is expected.

Applications should include a CV, relevant course records, and a motivation letter describing the applicant's interest in the subject and their assets for the project.

Working conditions and environment: The internship typically runs from March to August. Unless otherwise specified, interns receive the standard French stipend (*gratification de stage*). The work will take place in the MATH-EXP team (*Experimental Mathematics*) at the Inria Saclay Research Center (Turing Building, Palaiseau), where interns are expected to work on site on a daily basis. The team has long-term international collaborations with colleagues in Austria, Canada, China, Germany, Spain, and beyond.

PhD opportunity: A successful internship may naturally lead to a PhD project, potentially in the framework of an international collaboration (Vancouver). Funding and details will be discussed in due time.

References

- [1] Hadrien Brochet, Frédéric Chyzak, and Pierre Lairez. *Faster multivariate integration in D -modules*. 40 pages. 2025. arXiv: [2504.12724](#).
- [2] Frédéric Chyzak and Marni Mishna. *Differential equations satisfied by generating functions of 5-, 6-, and 7-regular labelled graphs: a reduction-based approach*. 34 pages. Accepted for publication in *Combinatorial Theory*. 2025. arXiv: [2406.04753](#).