Applications of Computer Algebra – ACA 2023
Warsaw, Poland, | July 17-21, 2023
Session *"D-Finite Functions and Beyond: Algorithms, Combinatorics, and Arithmetic"*

Reduction Based Creative Telescoping for Definite Summation of D-finite Functions: the Lagrange Identity Approach

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Creative telescoping is an algorithmic method initiated by Zeilberger [1] to compute definite sums and integrals. In the context of summation with regard to the variable t, given a summand $F(t, x_1, \ldots, x_m)$, where each x_i is a variable with an associated linear operator ∂_i (generally, differentiation or shift or q-shift operator), the goal is to construct identities of the form

$$\sum_{\boldsymbol{\alpha}} c_{\boldsymbol{\alpha}}(x_1, \ldots, x_m)\boldsymbol{\partial}^{\boldsymbol{\alpha}}(F) = G(t+1, x_1, \ldots, x_m) - G(t, x_1, \ldots, x_m), \tag{1}$$

where the sum is over a finite number of multi-indices $\boldsymbol{\alpha}$ and we use the multi-exponent notation $\boldsymbol{\partial}^{\boldsymbol{\alpha}} = \partial_1^{\alpha_1} \cdots \partial_m^{\alpha_m}$. Such an identity can in many applications be summed over t. Its right-hand side telescopes by design. Since the coefficients $c_{\boldsymbol{\alpha}}$ do not depend on the variable t, the left-hand side results in an operator applied to the definite sum of F. From there, other algorithms can be applied to compute information on the sum. The left-hand side of (1) is called a *telescoper* of F and the function G in the right-hand side is the corresponding *certificate*. Over the years, efficiency issues have led to the development of creative telescoping algorithms based on reductions. They avoid the computation of potentially large certificates and they compute telescopers in a more incremental fashion. In 2018 Bostan-Chyzak-Lairez-Salvy [2] published a reduction based algorithm for computing integrals of arbitrary D-finite functions. It was adapted to the summation case by van der Hoeven [4].

In my talk I will describe a new reduction based creative telescoping algorithm that is an adaption of the two previous ones. It computes telescopers for definite sums of D-finite functions as well as the associated certificates in a compact form. The algorithm relies on a discrete analogue of the generalized Hermite reduction introduced in [2] or equivalently, a generalization of the Abramov-Petkovšek reduction [3]. In contrast to van der Hoeven's algorithm, ours always returns the minimal order telescopers.

Keywords
Creative Telescoping, Symbolic Summation, D-finite Functions, Lagrange Identity, Hermite Reduction

References

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