

Linear recurrence sequences in the OEIS

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As of spring 2023, the On-Line Encyclopedia of Integer Sequences (OEIS) contains about 360 000 sequences [1]. Referencing an invited talk by Bruno Salvy at ISSAC 2005 [2], it is frequently stated that around 25% of the sequences in the OEIS satisfy a linear recurrence with polynomial coefficients. Using different guessing techniques we try to verify this claim and additionally give an estimate for the number of sequences satisfying a linear recurrence with constant coefficients. Furthermore, we study how this ratio changed over the past two decades and investigate the orders and degrees (in the case of polynomial coefficients) of the guessed recurrences.

Automatically proving positivity of a sequence which satisfies a linear recurrence is, in general, a difficult task [3]. Several algorithms are known which can be used to prove positivity for certain classes of these sequences where the recurrences have only constant coefficients. We take some of the sequences from the OEIS as a test set to examine how powerful these algorithms are [4].

Keywords

Recurrences, Guessing, OEIS, Positivity

References

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