Applications of Computer Algebra – ACA 2023
Warsaw, Poland, | July 17-21, 2023
Session *"D-Finite Functions and Beyond: Algorithms, Combinatorics, and Arithmetic"*

How a linear recurrence problem inspired a solution in algebraic geometry

Catherine St-Pierre[1,2] [catherine.st-pierre@uwaterloo.ca]

[1] Cheriton School of Computer Science, University of Waterloo Waterloo, Ontario, Canada
[2] University Paris-Saclay, Palaiseau, France

We review existing techniques to find terms in linearly recurrent sequences, such as Fiduccia's algorithm [1], and we focus on what can be done in some specific cases, such as when the recurrence is not square-free [2]. We rediscuss a map inspired by van der Hoeven and Lecerf [3] and how it ended up inspiring a method we use to address different problems arising from algebraic geometry or algebra, *e.g.* finding high powers of matrices [4]. We present some of the general lines of one of our recent works in the context of bivariate Gröbner bases [5], which is tailored to address the question of the local structure of the intersection of plane curves. In particular, we discuss the interest in moving the primary component to the origin and how it arises from a similar approach to what we use for sequences.

Keywords
Linear recurrence, algebraic geometry

References
[1] C. M. FIDUCCIA, An efficient formula for linear recurrences. *SIAM Journal on Computing*, 14(1), 106–112 (1985).
[2] S. G. HYUN; S. MELCZER; É. SCHOST; C. ST-PIERRE, Change of basis for m-primary ideals in one and two variables. In *Proceedings of the 2019 on International Symposium on Symbolic and Algebraic Computation*, 227–234 (2019).
[3] J. VAN DER HOEVEN; G. LECERF, Composition modulo powers of polynomials. In *Proceedings of the 2017 ACM on International Symposium on Symbolic and Algebraic Computation*, 445–452 (2017).
[4] M. GIESBRECHT, Nearly optimal algorithms for canonical matrix forms. *SIAM Journal on Computing*, 24(5), 948–969 (1995).
[5] É. SCHOST; C. ST-PIERRE, Newton iteration for lexicographic Gröbner bases in two variables. Preprint arXiv:2302.03766 (2023).