Same integrals as in Demo-redct, done with C. Koutschan's CreativeTelescoping

```
<< RISC`HolonomicFunctions`
```

HolonomicFunctions Package version 1.7.3 (21-Mar-2017)
written by Christoph Koutschan


```
--> Type  ?HolonomicFunctions  for help.
```

```
TimeBound = 60 * 60;
```

```
Test[f_, var_, rest_] := TimeConstrained[
  Timing[CreativeTelescoping[ Annihilator[f, Union[{var}, rest]], var, rest]],
  TimeBound]
```

Examples from C. Koutschan's Examples11.nb

I. Easy ones

```
st = TimeUsed[];
```

```
Test[(LegendreP[2 k + 1, x] / x) ^2, Der[x], {S[k]}]
```

$$\left\{0.560919, \left\{\{-S_k + 1\}, \left\{\frac{x^2 - x^4}{2\ (1 + k)\ (3 + 2 k)} D_x + \frac{x}{5 + 4 k} S_k + \frac{4 x + 3 k x - 5 x^3 - 4 k x^3}{(1 + k)\ (5 + 4 k)}\right\}\right\}\right\}$$

```
Test[Sqrt[1 - m * u^2] ^ (2 j - 1) / Sqrt[1 - u^2], Der[u], {S[j]}]
```

$$\{0.06082, \{\{(-3 - 2 j)\ S_j^2 + (4 + 4 j - 2 m - 2 j m)\ S_j + (-1 - 2 j + m + 2 j m)\}, \{m\ (u - u^3 - m u^3 + m u^5)\}\}\}$$

```
Test[ArcCos[x / Sqrt[(a + b) * x - a * b]], Der[x], {Der[a], Der[b]}]
```

$$\Big\{0.631294, \Big\{\Big\{(a^2 - b^2)\, D_b + (3\,a + b),\ (a^2 - b^2)\, D_a + (-a - 3\,b)\Big\},$$
$$\Big\{\frac{4\,(a^3\,b^2 - 2\,a^3\,b\,x - 2\,a^2\,b^2\,x + a^3\,x^2 + 3\,a^2\,b\,x^2 + a\,b^2\,x^2 - a^2\,x^3 - a\,b\,x^3)}{2\,a\,b - a\,x - b\,x}\,D_x + (a^2 + a\,b - 3\,a\,x - b\,x),$$
$$-\frac{4\,(a^2\,b^3 - 2\,a^2\,b^2\,x - 2\,a\,b^3\,x + a^2\,b\,x^2 + 3\,a\,b^2\,x^2 + b^3\,x^2 - a\,b\,x^3 - b^2\,x^3)}{2\,a\,b - a\,x - b\,x}\,D_x + (-a\,b - b^2 + a\,x + 3\,b\,x)\Big\}\Big\}\Big\}$$

```
Test[u^(2 m) / Sqrt[1 - u^2], Der[u], {S[m]}]
```

$$\{0.03256, \{\{(-2 - 2\,m)\,S_m + (1 + 2\,m)\}, \{-u + u^3\}\}\}$$

```
Test[1 / (x^4 + 2 a x^2 + 1)^(m + 1), Der[x], {S[m], Der[a]}]
```

$$\Big\{0.084665, \Big\{\Big\{(-4 - 4\,m)\,S_m + 2\,a\,D_a + (3 + 4\,m),\ (-4 + 4\,a^2)\,D_a^2 + (12\,a + 8\,a\,m)\,D_a + (3 + 4\,m)\Big\},$$
$$\Big\{x, \frac{-3\,x - 4\,m\,x - 2\,a\,x^3 - 4\,a\,m\,x^3 + x^5}{1 + 2\,a\,x^2 + x^4}\Big\}\Big\}\Big\}$$

```
Test[LegendreP[2 n, u] / Sqrt[1 - u^2], Der[u], {S[n]}]
```

$$\Big\{0.247012,$$
$$\Big\{\Big\{(4 + 8\,n + 4\,n^2)\,S_n + (-1 - 4\,n - 4\,n^2)\Big\}, \Big\{-\frac{2\,(1 + n)\,(-1 + u^2)}{u}\,S_n + \frac{1 + 2\,n - u^2 - 2\,n\,u^2}{u}\Big\}\Big\}\Big\}$$

```
Test[Sin[m * x] * Sin[n * x], Der[x], {S[m], S[n]}]
```

Annihilator: The expression Sin[m*x]w.r.t. {S[m]} is not recognized to be ∂–finite. The result might not generate a zero–dimensional ideal.

Annihilator: The expression Sin[n*x]w.r.t. {S[n]} is not recognized to be ∂–finite. The result might not generate a zero–dimensional ideal.

$$\{0.796215, \{\{\}, \{\}\}\}$$

```
Test[Exp[-u (a + 1)] * Log[u], Der[a], {Der[u]}]
```

$$\Big\{0.083013, \Big\{\{1\}, \Big\{\frac{1}{u}\Big\}\Big\}\Big\}$$

Victor Moll's integral: special case s=4.

```
ff = (x^2 / (x^4 + 2 a x^2 + 1))^r * (x^2 + 1) / x^2 / (x^s + 1) /. s → 4
```

$$\frac{(1 + x^2)\left(\frac{x^2}{1 + 2\,a\,x^2 + x^4}\right)^r}{x^2\,(1 + x^4)}$$

```
Test[ff, Der[x], {Der[a], S[r]}]
```

$$\{0.191629, \{\{D_a + 2\,r\,S_r,\ (64\,a - 64\,a^3 + 96\,a\,r - 96\,a^3\,r + 32\,a\,r^2 - 32\,a^3\,r^2)\,S_r^3 + (-32 - 16\,a + 64\,a^2 - 48\,r - 16\,a\,r + 112\,a^2\,r - 16\,r^2 + 48\,a^2\,r^2)\,S_r^2 + (8 - 10\,a + 8\,r - 40\,a\,r - 24\,a\,r^2)\,S_r + (-3 + 4\,r + 4\,r^2)\},\ \{0,\ (-3\,x - 2\,r\,x - 5\,x^3 - 2\,a\,x^3 - 2\,r\,x^3 - 2\,x^5 - 6\,a\,x^5 - 6\,x^7 - 2\,a\,x^7 + x^9 - 6\,a\,x^9 + 2\,r\,x^9 - x^{11} + 2\,r\,x^{11}) \Big/ \left((1 + x^2)\,(1 + 2\,a\,x^2 + x^4)^2\right)\}\}\}$$

```
Test[ArcSinh[x] / Sqrt[x^2 + 1] * E^(-z * x), Der[x], {Der[z]}]
```

$$\left\{0.194286, \left\{\{z\,D_z^2 + D_z + z\}, \left\{\frac{1 + x^2}{z}\,D_x + \frac{x + 2\,z + 2\,x^2\,z}{z}\right\}\right\}\right\}$$

```
Test[z / 2 * ArcSinh[x]^2 * E^(-z * x), Der[x], {Der[z]}]
```

$$\left\{0.348777, \left\{\{z\,D_z^2 + D_z + z\}, \left\{\frac{1 + x^2}{z^2}\,D_x^2 + \frac{x + 3\,z + 3\,x^2\,z}{z^2}\,D_x + 3\,(1 + x^2)\right\}\right\}\right\}$$

```
Test[x * ArcSinh[x] / Sqrt[x^2 + 1] * E^(-z * x) / 2, Der[x], {Der[z]}]
```

$$\left\{0.244387, \left\{\{z^2\,D_z^2 + z\,D_z + (-1 + z^2)\}, \left\{\frac{2 + 2\,x^2 + x\,z + x^3\,z}{x\,z}\,D_x + \frac{-2 + 2\,x\,z + 3\,x^3\,z + 2\,x^2\,z^2 + 2\,x^4\,z^2}{x^2\,z}\right\}\right\}\right\}$$

```
Test[ArcSinh[x] / (1 + x^2)^(n + 1), Der[x], {S[n]}]
```

$$\left\{0.160012, \left\{\{(-2 - 2\,n)\,S_n + (1 + 2\,n)\}, \left\{\frac{1 + x^2}{1 + 2\,n}\,D_x + \frac{3\,x + 4\,n\,x}{1 + 2\,n}\right\}\right\}\right\}$$

```
annLommel = ToOrePolynomial[
  {S[n] - 1, Der[x] ** (x^2 * Der[x]^2 + x * Der[x] + x^2)}, OreAlgebra[Der[x], S[n]]]
```

$$\{S_n - 1,\ x^2\,D_x^3 + 3\,x\,D_x^2 + (1 + x^2)\,D_x + 2\,x\}$$

```
integrand =
 DFiniteTimes[Annihilator[x^n * BesselJ[n, x], {Der[x], S[n]}], annLommel]
```

$$\{S_n^2 + (-2 - 2\,n)\,S_n + x^2,\ x^3\,D_x^3 + 3\,x^2\,D_x^2\,S_n + (3\,x^2 - 6\,n\,x^2)\,D_x^2 + (3\,x - 6\,n\,x)\,D_x\,S_n + (x - 6\,n\,x + 12\,n^2\,x - 2\,x^3)\,D_x + 4\,n^2\,S_n + (-8\,n^3 - 2\,x^2 + 2\,n\,x^2)\}$$

```
ct = CreativeTelescoping[integrand, Der[x], S[n]]
```

$$\{\{(4 + 2\,n)\,S_n^2 + (-11 - 18\,n - 8\,n^2)\,S_n + (1 + 6\,n + 12\,n^2 + 8\,n^3)\},\ \{-x^3\,D_x^2 - 3\,x^2\,D_x\,S_n + 6\,n\,x^2\,D_x + 3\,(x + 2\,n\,x)\,S_n + (-x - 6\,n\,x - 12\,n^2\,x + 2\,x^3)\}\}$$

```
Test[(1 + x t + t^2)^(-2), Der[t], {Der[x]}]
```

$$\{0.026689, \{\{(-4 + x^2)\,D_x + 3\,x\}, \{2 + t\,x\}\}\}$$

```
Test[1 / (1 + x^2)^n, Der[x], { S[n]}]
```

$$\{0.028706, \{\{-2\,n\,S_n + (-1 + 2\,n)\}, \{x\}\}\}$$

```
Test[x^(m-1) * Exp[-c * x - b * x^2] * Sin[a * x],
 Der[x], {S[m], Der[a], Der[b], Der[c]}]
```

$$\{0.598917,\ \{\{a\, D_a + 2\, b\, D_b + c\, D_c + m,\ S_m + D_c,\ D_c^2 + D_b,\ 4\, b^2\, D_b^2 + 4\, b\, c\, D_b\, D_c + \left(-a^2 + 6\, b - c^2 + 4\, b\, m\right) D_b + (2\, c + 2\, c\, m)\, D_c + \left(m + m^2\right)\},\ \{-x,\ 0,\ 0,\ x^2\, D_x - 2\left(m\, x - c\, x^2 - 2\, b\, x^3\right)\}\}\}$$

```
Test[ArcTan[p * x] / (1 + p^2 * x), Der[x], {Der[p]}]
```

$$\{0.293415,\ \{\{\left(p^2 + p^4\right) D_p^2 + \left(4\, p + 6\, p^3\right) D_p + \left(2 + 6\, p^2\right)\},\ \{\frac{1 - p^4\, x^2}{p^2}\, D_x - \frac{2\left(-1 + x + 2\, p^2\, x - p^2\, x^2 + p^4\, x^2\right)}{1 + p^2\, x}\}\}\}$$

```
Test[BesselJ[m, a * x] * BesselJ[n, b * x], Der[x], {S[n], S[m], Der[a], Der[b]}]
```

$$\{2.18831,\ \{\{a\, D_a + b\, D_b + 1,\ \left(a^2\, b^2 - b^4\right) D_b^2 + \left(a^2\, b - 3\, b^3\right) D_b + \left(-b^2 + b^2\, m^2 - a^2\, n^2\right),$$
$$\left(a^2\, b - b^3\right) S_m\, D_b + (a\, b\, m - a\, b\, n)\, S_n + \left(b^2\, m - a^2\, n\right) S_m,\ \left(a^2 + 2\, a^2\, m + a^2\, m^2 - a^2\, n^2\right) S_m^2 +$$
$$\left(-2\, a^2\, b + 2\, b^3 - 2\, a^2\, b\, m + 2\, b^3\, m\right) D_b + \left(-a^2 + 2\, b^2 - 2\, a^2\, m + 4\, b^2\, m - a^2\, m^2 + 2\, b^2\, m^2 - a^2\, n^2\right),$$
$$\left(-a^2\, b + b^3\right) S_n\, D_b + \left(-a^2 + b^2 + b^2\, m - a^2\, n\right) S_n + (a\, b\, m - a\, b\, n)\, S_m,$$
$$(a\, b + a\, b\, m + a\, b\, n)\, S_n\, S_m + \left(a^2\, b - b^3\right) D_b + \left(-b^2 - b^2\, m - a^2\, n\right),$$
$$\left(-b^2 + b^2\, m^2 - 2\, b^2\, n - b^2\, n^2\right) S_n^2 + \left(2\, a^2\, b - 2\, b^3 + 2\, a^2\, b\, n - 2\, b^3\, n\right) D_b +$$
$$\left(-b^2 + b^2\, m^2 - 2\, a^2\, n - 2\, a^2\, n^2 + b^2\, n^2\right)\},$$
$$\{-x,\ a\, b^2\, x^2\, S_m - b^3\, x^2\, S_n - b\,(-b\, x + b\, m\, x - b\, n\, x),\ b^2\, x\, S_m - a\, b\, x\, S_n,$$
$$-2\, a\,(b + b\, m)\, x\, S_m\, S_n + 2\, a\,(1 + m)\,(1 + m + n)\, S_m - 2\left(b^2\, x + b^2\, m\, x\right),\ a\, b\, x\, S_m - b^2\, x\, S_n,$$
$$a\, b\, x\, S_m\, S_n + b^2\, x,\ 2\, b\,(a + a\, n)\, x\, S_m\, S_n - 2\, b\,(1 + n)\,(1 + m + n)\, S_n + 2\, b^2\,(x + n\, x)\}\}\}$$

```
Test[Log[(Sqrt[x^2 + a^2] + x) / (Sqrt[x^2 + a^2] - x)] *
  BesselJ[0, b * x] / Sqrt[x^2 + a^2], Der[x], {Der[a], Der[b]}]
```

$$\{1.49979,\ \{\{-a\, D_a + b\, D_b,\ b^2\, D_b^3 + 3\, b\, D_b^2 + \left(1 - a^2\, b^2\right) D_b - a^2\, b\},\ \{-x,\ \frac{a^2\, x + x^3}{2\, b}\, D_x^2 + \frac{a^2}{2\, x}\, D_b + \frac{3\, x^2}{2\, b}\, D_x + \frac{x + 3\, a^2\, b^2\, x + 3\, b^2\, x^3}{2\, b}\}\}\}$$

```
Test[(1 - x^2)^(nu - 1/2) * GegenbauerC[m, nu, x] * GegenbauerC[n, nu, x],
 Der[x], {S[n], S[m], S[nu]}]
```

$$\{1.25848,\ \{\{1\},\ \{\frac{-1 - m}{(m - n)\,(m + n + 2\, nu)}\, S_m + \frac{1 + n}{(m - n)\,(m + n + 2\, nu)}\, S_n + \frac{x}{m + n + 2\, nu}\}\}\}$$

```
Test[(x (2 a - x)) ^ (nu - 1 / 2) * GegenbauerC[n, nu, x / a - 1] * E ^ (-b * x),
 Der[x], {Der[a], Der[b], S[n], S[nu]}]
```

$$\Big\{1.00722, \Big\{\Big\{\left(-a - 2\,a\,n - a\,n^2 - 2\,a\,nu - 2\,a\,n\,nu\right) S_n - 2\,b\,nu\,S_{nu},$$
$$\left(b\,n + b\,n^2 + 2\,b\,nu + 4\,b\,n\,nu + 4\,b\,nu^2\right) D_b - 2\,b^2\,nu\,S_{nu} + \left(a\,b\,n - n^2 + a\,b\,n^2 - n^3 + 2\,a\,b\,nu - 2\,n\,nu + 4\,a\,b\,n\,nu - 4\,n^2\,nu + 4\,a\,b\,nu^2 - 4\,n\,nu^2\right),$$
$$\left(a\,n + a\,n^2 + 2\,a\,nu + 4\,a\,n\,nu + 4\,a\,nu^2\right) D_a - 2\,b^2\,nu\,S_{nu} + \big(a\,b\,n - n^2 + a\,b\,n^2 - n^3 + 2\,a\,b\,nu - 4\,n\,nu + 4\,a\,b\,n\,nu - 6\,n^2\,nu - 4\,nu^2 + 4\,a\,b\,nu^2 - 12\,n\,nu^2 - 8\,nu^3\big),$$
$$\left(4\,b^2\,nu + 4\,b^2\,nu^2\right) S_{nu}^2 + \big(24\,nu + 44\,n\,nu + 24\,n^2\,nu + 4\,n^3\,nu + 64\,nu^2 + 76\,n\,nu^2 + 20\,n^2\,nu^2 + 56\,nu^3 + 32\,n\,nu^3 + 16\,nu^4\big)\,S_{nu} +$$
$$\big(-6\,a^2\,n - 11\,a^2\,n^2 - 6\,a^2\,n^3 - a^2\,n^4 - 12\,a^2\,nu - 44\,a^2\,n\,nu - 36\,a^2\,n^2\,nu - 8\,a^2\,n^3\,nu - 44\,a^2\,nu^2 - 72\,a^2\,n\,nu^2 - 24\,a^2\,n^2\,nu^2 - 48\,a^2\,nu^3 - 32\,a^2\,n\,nu^3 - 16\,a^2\,nu^4\big)\Big\},$$
$$\Big\{-2\,nu\,S_{nu},\ \frac{2\,nu\,(1 - a\,b + n + 2\,nu + b\,x)}{a - x}\,S_{nu} + \frac{1}{a - x}\big(-2\,a\,n\,x - 2\,a\,n^2\,x - 4\,a\,nu\,x - 8\,a\,n\,nu\,x - 8\,a\,nu^2\,x + n\,x^2 + n^2\,x^2 + 2\,nu\,x^2 + 4\,n\,nu\,x^2 + 4\,nu^2\,x^2\big),$$
$$\frac{2\,nu\,(1 - a\,b + n + 2\,nu + b\,x)}{a - x}\,S_{nu} + \frac{1}{a - x}\left(-a\,n\,x - a\,n^2\,x - 2\,a\,nu\,x - 4\,a\,n\,nu\,x - 4\,a\,nu^2\,x\right),$$
$$-\frac{1}{a - x}\,2\,nu\,\big(-3\,a^2 - a^3\,b - 4\,a^2\,n - a^2\,n^2 - 2\,a^2\,nu - 2\,a^3\,b\,nu - 2\,a^2\,n\,nu + 12\,a\,x - 3\,a^2\,b\,x + 16\,a\,n\,x - 4\,a^2\,b\,n\,x + 4\,a\,n^2\,x + 20\,a\,nu\,x - 2\,a^2\,b\,nu\,x + 12\,a\,n\,nu\,x + 8\,a\,nu^2\,x - 6\,x^2 + 6\,a\,b\,x^2 - 8\,n\,x^2 + 6\,a\,b\,n\,x^2 - 2\,n^2\,x^2 - 10\,nu\,x^2 + 6\,a\,b\,nu\,x^2 - 6\,n\,nu\,x^2 - 4\,nu^2\,x^2 - 2\,b\,x^3 - 2\,b\,n\,x^3 - 2\,b\,nu\,x^3\big)\,S_{nu} +$$
$$\frac{1}{a - x}\big(6\,a^3\,n\,x - 2\,a^4\,b\,n\,x + 8\,a^3\,n^2\,x - 2\,a^4\,b\,n^2\,x + 2\,a^3\,n^3\,x + 12\,a^3\,nu\,x - 4\,a^4\,b\,nu\,x + 32\,a^3\,n\,nu\,x - 8\,a^4\,b\,n\,nu\,x + 12\,a^3\,n^2\,nu\,x + 32\,a^3\,nu^2\,x - 8\,a^4\,b\,nu^2\,x + 24\,a^3\,n\,nu^2\,x + 16\,a^3\,nu^3\,x - 3\,a^2\,n\,x^2 + 3\,a^3\,b\,n\,x^2 - 4\,a^2\,n^2\,x^2 + 3\,a^3\,b\,n^2\,x^2 - a^2\,n^3\,x^2 - 6\,a^2\,nu\,x^2 + 6\,a^3\,b\,nu\,x^2 - 16\,a^2\,n\,nu\,x^2 + 12\,a^3\,b\,n\,nu\,x^2 - 6\,a^2\,n^2\,nu\,x^2 - 16\,a^2\,nu^2\,x^2 + 12\,a^3\,b\,nu^2\,x^2 - 12\,a^2\,n\,nu^2\,x^2 - 8\,a^2\,nu^3\,x^2 - a^2\,b\,n\,x^3 - a^2\,b\,n^2\,x^3 - 2\,a^2\,b\,nu\,x^3 - 4\,a^2\,b\,n\,nu\,x^3 - 4\,a^2\,b\,nu^2\,x^3\big)\Big\}\Big\}\Big\}$$

```
Test[ChebyshevT[n, 1 - x^2 y] / Sqrt[1 - x^2], Der[x], {S[n], Der[y]}]
```

$$\Big\{0.504911, \Big\{\Big\{\left(2\,n + 2\,n^2\right) S_n + \left(-2\,y - 4\,n\,y + y^2 + 2\,n\,y^2\right) D_y + \left(-2\,n - 2\,n^2 + n\,y + 2\,n^2\,y\right),$$
$$\left(2\,y - y^2\right) D_y^2 + \left(2 - y\right) D_y + n^2\Big\},$$
$$\Big\{-\frac{n\left(-1 + x^2\right)}{x}\,S_n + \frac{n\left(-1 + x^2\right)}{x},\ \frac{n\left(-1 + x^2\right)}{x\,y\left(-2 + x^2\,y\right)}\,S_n + \frac{n\left(1 - x^2 - x^2\,y + x^4\,y\right)}{x\,y\left(-2 + x^2\,y\right)}\Big\}\Big\}\Big\}$$

```
Test[x^(r - 1) * (1 - x)^(s - 1) * Hypergeometric2F1[a, b, c, x],
 Der[x], {S[a], S[b], S[c], S[r], S[s]}]
```

$$\Big\{1.10722, \Big\{\Big\{S_r + S_s - 1,\ \left(a\,b\,c - a\,c^2 - b\,c^2 + c^3 - a\,b\,r + a\,c\,r + b\,c\,r - c^2\,r\right) S_c +$$
$$\left(-a\,b\,c + a\,c\,r + b\,c\,r - c\,r^2 + a\,c\,s + b\,c\,s - 2\,c\,r\,s - c\,s^2\right) S_s +$$
$$\left(a\,c^2 + b\,c^2 - c^3 - a\,c\,r - b\,c\,r + c^2\,r - a\,c\,s - b\,c\,s + c\,r\,s + c\,s^2\right),$$
$$\left(-b - a\,b - b^2 + b\,c + b\,s\right) S_b + \left(a\,b - a\,r - b\,r + r^2 - a\,s - b\,s + 2\,r\,s + s^2\right) S_s +$$
$$\left(b + b^2 - b\,c + a\,s - r\,s - s^2\right),\ \left(-a - a^2 - a\,b + a\,c + a\,s\right) S_a +$$
$$\left(a\,b - a\,r - b\,r + r^2 - a\,s - b\,s + 2\,r\,s + s^2\right) S_s + \left(a + a^2 - a\,c + b\,s - r\,s - s^2\right),$$
$$\left(1 - a - b + a\,b + 2\,r - a\,r - b\,r + r^2 + 2\,s - a\,s - b\,s + 2\,r\,s + s^2\right) S_s^2 +$$
$$\left(-1 + a + b - a\,b - r + a\,r + b\,r - c\,r - 2\,s + 2\,a\,s + 2\,b\,s - c\,s - 2\,r\,s - 2\,s^2\right) S_s +$$
$$\left(-a\,s - b\,s + c\,s + s^2\right)\Big\},$$
$$\Big\{0,\ (a - c)\,(-b + c)\,\left(-x + x^2\right) S_c + \left(-c^2\,x + c\,r\,x + c\,s\,x + c^2\,x^2 - c\,r\,x^2 - c\,s\,x^2\right),$$
$$-\frac{(a - c)\,(-b + c)\,x^2}{c} S_c + \left(a\,x + b\,x - r\,x - s\,x - c\,x^2 + r\,x^2 + s\,x^2\right),$$
$$-\frac{(a - c)\,(-b + c)\,x^2}{c} S_c + \left(a\,x + b\,x - r\,x - s\,x - c\,x^2 + r\,x^2 + s\,x^2\right),$$
$$\frac{(a - c)\,(-b + c)\,\left(-x^2 + x^3\right)}{c} S_c +$$
$$\left(-x + c\,x - r\,x + 2\,x^2 - 2\,c\,x^2 + 2\,r\,x^2 + s\,x^2 - x^3 + c\,x^3 - r\,x^3 - s\,x^3\right)\Big\}\Big\}\Big\}$$

```
Test[(z / 2)^n / Gamma[n + 1 / 2] / Gamma[1 / 2] * (1 - t^2)^(n - 1 / 2) * Cos[z t],
 Der[t], {Der[z], S[n]}]
```

$$\Big\{0.394906, \Big\{\Big\{z\,D_z + z\,S_n - n,\ z\,S_n^2 + (-2 - 2\,n)\,S_n + z\Big\},\ \Big\{-\frac{\left(-1 + t^2\right) z}{(1 + 2\,n)\,t} D_z + \frac{-n + n\,t^2}{(1 + 2\,n)\,t},$$
$$\frac{\left(1 - 2\,t^2 + t^4\right) z^2}{(1 + 2\,n)\,(3 + 2\,n)\,t} D_z - \frac{\left(n - 3\,t^2 - 4\,n\,t^2 + 3\,t^4 + 3\,n\,t^4\right) z}{(1 + 2\,n)\,(3 + 2\,n)\,t}\Big\}\Big\}\Big\}$$

```
Test[t^(-n - 1) * Exp[t - z^2 / (4 t)], Der[t], {S[n], Der[z]}]
```

$$\Big\{0.067646, \Big\{\Big\{z\,S_n + 2\,D_z,\ z\,D_z^2 + (1 + 2\,n)\,D_z + z\Big\},\ \{0, -z\}\Big\}\Big\}$$

```
Test[Sin[z t] (1 - t^2)^(n + 1 / 2), Der[t], {S[n], Der[z]}]
```

$$\Big\{0.351968, \Big\{\Big\{z\,S_n + (3 + 2\,n)\,D_z,\ z\,D_z^2 + (3 + 2\,n)\,D_z + z\Big\},\ \Big\{\frac{1 - t^2}{t} D_z,\ \frac{1 - t^2}{t} D_z\Big\}\Big\}\Big\}$$

```
Test[Exp[-z t] (1 + t^2)^(n - 1 / 2), Der[t], {S[n], Der[z]}]
```

$$\Big\{0.068952, \Big\{\Big\{z\,S_n + (1 + 2\,n)\,D_z,\ z\,D_z^2 + (1 + 2\,n)\,D_z + z\Big\},\ \Big\{1 + t^2,\ 1 + t^2\Big\}\Big\}\Big\}$$

```
Test[BesselJ[n, b t] * Exp[-p^2 * t^2] * t^(n + 1), Der[t], {Der[b], S[n], Der[p]}]
```

$$\Big\{0.349049, \Big\{\Big\{2\,p^3\,D_p + \left(-b^2 + 4\,p^2 + 4\,n\,p^2\right),\ 2\,p^2\,S_n - b,\ 2\,b\,p^2\,D_b + \left(b^2 - 2\,n\,p^2\right)\Big\},$$
$$\Big\{\frac{b}{t} S_n - 2\,p^2\,t,\ \frac{1}{t} S_n,\ -\frac{b}{t} S_n\Big\}\Big\}\Big\}$$

```
Test[t^n * BesselY[n, a t] / (t^2 + k^2), Der[t], {S[n], Der[a], Der[k]}]
```

$$\left\{0.564635, \left\{\left\{a D_a - k D_k + (-1+n),\ a S_n + k D_k + (1-2n),\ k^2 D_k^3 + \left(5k - 2kn\right) D_k^2 + \left(4 - a^2 k^2 - 4n\right) D_k - 2a^2 k\right\}, \left\{-t, t, \frac{2akt}{k^2+t^2} S_n + \frac{2k\left(k^2 t - 3t^3\right)}{\left(k^2+t^2\right)^2}\right\}\right\}\right\}$$

```
Test[Exp[-p^2 * t^2] * BesselJ[0, a t] * BesselY[0, a t] * t, Der[t], {Der[a], Der[p]}]
```

$$\left\{0.472175, \left\{\left\{a D_a + p D_p + 2,\ p^4 D_p^2 + \left(-2a^2 p + 5p^3\right) D_p + \left(-2a^2 + 4p^2\right)\right\}, \left\{-t, \frac{t}{2} D_t^2 + \frac{1}{2}\left(-1 + 6p^2 t^2\right) D_t + \frac{1 + 4a^2 t^2 - 6p^2 t^2 + 12 p^4 t^4}{2t}\right\}\right\}\right\}$$

```
Test[z^(n + 1) * BesselI[n, z], Der[z], {S[n]}]
```

$$\left\{0.110076, \left\{\{1\}, \left\{-\frac{1}{z} S_n\right\}\right\}\right\}$$

```
Test[t^(a - 1) * (1 - t)^(b - 1), Der[t], {S[a], S[b]}]
```

$$\left\{0.052316, \left\{\{(-a-b) S_b + b,\ (-a-b) S_a + a\}, \left\{t - t^2, -t + t^2\right\}\right\}\right\}$$

```
Test[Exp[-a t] * StruveH[0, t], Der[t], {Der[a]}]
```

$$\left\{0.210666, \left\{\left\{\left(-1-a^2\right) D_a - a\right\}, \left\{\frac{t}{a} D_t^2 + \frac{1+3at}{a} D_t + \frac{a + t + 3a^2 t}{a}\right\}\right\}\right\}$$

```
Test[AiryAi[2^(2/3) * (u^2 + x)], Der[u], {Der[x]}]
```

$$\left\{0.201503, \left\{\left\{D_x^3 - 4x D_x - 2\right\}, \{-2u\}\right\}\right\}$$

```
TimeUsed[] - st
```

16.1223

2. A longer one

```
Test[2 / Pi * BesselJ[m + n, 2 z t] * ChebyshevT[m - n, t] / Sqrt[1 - t^2],
 Der[t], {S[m], S[n], Der[z]}]
```

$$\left\{18.6877, \left\{\left\{z S_m + z S_n + z D_z + (-m-n),\ 2z^2 S_n D_z + z^2 D_z^2 + 2z S_n + (z - 2nz) D_z + \left(-m^2 + n^2\right),\ z S_n^2 + (-2-2n) S_n + z,\ z^3 D_z^3 + 3z^2 D_z^2 + \left(2m^2 z - 2n^2 z\right) S_n + \left(z - m^2 z - 3n^2 z + 4z^3\right) D_z + \left(-2m^2 n + 2n^3 + 4z^2\right)\right\}, \left\{0, tz S_m - tz S_n, -\frac{1}{2t} S_m + \frac{-1+2t^2}{2t} S_n, -(mt - nt) z S_m + (mt - nt) z S_n + 4\left(-t + t^3\right) z^2\right\}\right\}\right\}$$

3. Examples that are hard for redct, or where it needs help

```
Test[GegenbauerC[l, λ, x] * GegenbauerC[m, λ, x] *
  GegenbauerC[n, λ, x] * (1 - x^2) ^ (λ - 1 / 2), Der[x], {S[m], S[n], S[l]}]
```

$$\{252.95, \{\{(1 - 2 l + l^2 - m^2 + 2 n - 2 l n + n^2 - 2 \lambda + 2 l \lambda - 2 m \lambda - 2 n \lambda) S_n + (-1 - 2 l - l^2 + m^2 + 2 n + 2 l n - n^2 + 2 \lambda + 2 l \lambda + 2 m \lambda - 2 n \lambda) S_l,$$

$$(1 - 2 l + l^2 + 2 m - 2 l m + m^2 - n^2 - 2 \lambda + 2 l \lambda - 2 m \lambda - 2 n \lambda) S_m + (-1 - 2 l - l^2 + 2 m + 2 l m - m^2 + n^2 + 2 \lambda + 2 l \lambda - 2 m \lambda + 2 n \lambda) S_l,$$

$$(-16 - 32 l - 24 l^2 - 8 l^3 - l^4 + 8 m^2 + 8 l m^2 + 2 l^2 m^2 - m^4 + 8 n^2 + 8 l n^2 + 2 l^2 n^2 + 2 m^2 n^2 - n^4 + 16 m \lambda + 16 l m \lambda + 4 l^2 m \lambda - 4 m^3 \lambda + 16 n \lambda + 16 l n \lambda + 4 l^2 n \lambda + 4 m^2 n \lambda + 4 m n^2 \lambda - 4 n^3 \lambda + 16 \lambda^2 + 16 l \lambda^2 + 4 l^2 \lambda^2 - 4 m^2 \lambda^2 + 8 m n \lambda^2 - 4 n^2 \lambda^2) S_l^2 +$$

$$(l^4 - 2 l^2 m^2 + m^4 - 2 l^2 n^2 - 2 m^2 n^2 + n^4 + 8 l^3 \lambda - 4 l^2 m \lambda - 8 l m^2 \lambda + 4 m^3 \lambda - 4 l^2 n \lambda - 4 m^2 n \lambda - 8 l n^2 \lambda - 4 m n^2 \lambda + 4 n^3 \lambda + 20 l^2 \lambda^2 - 16 l m \lambda^2 - 4 m^2 \lambda^2 - 16 l n \lambda^2 - 8 m n \lambda^2 - 4 n^2 \lambda^2 + 16 l \lambda^3 - 16 m \lambda^3 - 16 n \lambda^3)\},$$

$$\{(-1 - m) S_l S_m + (-l + n) S_l S_n + (1 + m) S_m S_n + (x + l x + m x - n x) S_l + (-1 + l - m - n) x S_n + (-l + n), (-l + m) S_l S_m + (-1 - n) S_l S_n + (1 + n) S_m S_n + (x + l x - m x + n x) S_l + (-1 + l - m - n) x S_m + (-l + m),$$

$$4 (1 + m) (1 + n) (1 + l + \lambda) S_l S_m S_n - \frac{1}{2 + l} 2 (1 + m) (1 + l + \lambda) (4 x + 4 l x + l^2 x - m^2 x + 4 n x + 2 l n x + n^2 x + 4 x \lambda + 2 l x \lambda - 2 m x \lambda + 2 n x \lambda) S_l S_m -$$

$$\frac{1}{2 + l} 2 (1 + n) (1 + l + \lambda) (4 x + 4 l x + l^2 x + 4 m x + 2 l m x + m^2 x - n^2 x + 4 x \lambda + 2 l x \lambda + 2 m x \lambda - 2 n x \lambda) S_l S_n +$$

$$\frac{1}{2 + l} 2 (1 + l + \lambda) (-4 - 6 l - 4 l^2 - l^3 + 2 m^2 + l m^2 + 2 n^2 + l n^2 + 8 x^2 + 12 l x^2 + 6 l^2 x^2 + l^3 x^2 + 4 m x^2 + 4 l m x^2 + l^2 m x^2 - 2 m^2 x^2 - l m^2 x^2 - m^3 x^2 + 4 n x^2 + 4 l n x^2 + l^2 n x^2 + 4 m n x^2 + 2 l m n x^2 + m^2 n x^2 - 2 n^2 x^2 - l n^2 x^2 + m n^2 x^2 - n^3 x^2 - 4 l \lambda - 2 l^2 \lambda + 4 m \lambda + 2 l m \lambda + 4 n \lambda + 2 l n \lambda + 8 x^2 \lambda + 8 l x^2 \lambda + 2 l^2 x^2 \lambda - 2 m^2 x^2 \lambda + 4 m n x^2 \lambda - 2 n^2 x^2 \lambda)$$

$$S_l + \frac{1}{2 + l} 2 (1 + m) (1 + l + \lambda) (2 l + l^2 - m^2 + n^2 + 4 \lambda + 2 l \lambda - 2 m \lambda + 2 n \lambda) S_m +$$

$$\frac{1}{2 + l} 2 (1 + n) (1 + l + \lambda) (2 l + l^2 + m^2 - n^2 + 4 \lambda + 2 l \lambda + 2 m \lambda - 2 n \lambda) S_n - \frac{1}{2 + l}$$

$$2 (1 + l + \lambda) (4 l x + 2 l^2 x + 2 l m x + l^2 m x - m^3 x + 2 l n x + l^2 n x + m^2 n x + m n^2 x - n^3 x + 8 x \lambda + 4 l x \lambda + 4 m x \lambda + 2 l m x \lambda - 2 m^2 x \lambda + 4 n x \lambda + 2 l n x \lambda + 4 m n x \lambda - 2 n^2 x \lambda)\}\}\}$$

```
Test[x BesselJ[1, a x] BesselI[1, a x] BesselY[0, x] BesselK[0, x], Der[x], {Der[a]}]
```

$$\Big\{45.0437, \Big\{\{a\,D_a+2\}, \Big\{-\frac{5a^2}{4\left(-1+a^4\right)x}D_a^2D_x^2+\frac{5a}{4\left(-1+a^4\right)}D_aD_x^3-\frac{3x}{8\left(-1+a^4\right)}D_x^4+\frac{a^3}{\left(-1+a^4\right)x^3}D_a^3+\frac{a^2}{\left(-1+a^4\right)x^2}D_a^2D_x-\frac{3a}{2\left(-1+a^4\right)x}D_aD_x^2-\frac{1}{2\left(-1+a^4\right)}D_x^3+\frac{5a^2}{2\left(-1+a^4\right)x^3}D_a^2+\frac{7a}{2\left(-1+a^4\right)x^2}D_aD_x-\frac{3}{8\left(-1+a^4\right)x}D_x^2-\frac{5a}{\left(-1+a^4\right)x^3}D_a+\frac{5}{8\left(-1+a^4\right)x^2}D_x+\frac{-5-12x^4-20a^4x^4}{8\left(-1+a^4\right)x^3}\Big\}\Big\}\Big\}$$

```
Test[x BesselJ[1, a x] BesselI[1, a x] BesselY[0, x] BesselK[0, x], Der[a], {Der[x]}]
```

$$\Big\{3.96648, \Big\{\{-x^3D_x^4-4x^2D_x^3-x\,D_x^2+D_x-4x^3\}, \Big\{-\frac{a^4}{x}D_a^3+4a^3D_a^2D_x-6a^2x\,D_aD_x^2+4a\,x^2D_x^3-\frac{8a^3}{x}D_a^2+12a^2D_aD_x-\frac{13a^2}{x}D_a+2a\,D_x-\frac{3a}{x}\Big\}\Big\}\Big\}$$

```
Test[(c + I * u * Sqrt[1 - c^2])^n / Sqrt[1 - u^2], Der[u], {S[n], Der[c]}]
```

$$\Big\{0.730564, \Big\{\{(1+n)\,S_n+\left(1-c^2\right)D_c+(-c-c\,n),\ \left(-1+c^2\right)D_c^2+2c\,D_c+\left(-n-n^2\right)\},\Big\{\frac{1-u^2}{u}S_n+\frac{-c+c\,u^2}{u},\ \frac{c\,n\left(-1+u^2\right)}{(-1+c)\,(1+c)\,u\left(-c^2-u^2+c^2u^2\right)}S_n-\left(n\left(-c^2+u^2-u^4+c^2u^4\right)\right)\Big/\left((-1+c)\,(1+c)\,u\left(-c^2-u^2+c^2u^2\right)\right)\Big\}\Big\}\Big\}$$

```
Test[Exp[x] * x^(-a / 2) / n! * Exp[-t] * t^(n + a / 2) * BesselJ[a, 2 (t * x) ^ (1 / 2)],
 Der[t], {S[n], S[a], Der[x]}]
```

$$\Big\{0.977198, \Big\{\{(-1-n)\,S_n+x\,D_x+(1+a+n-x),\ x\,D_x^2+(1+a-x)\,D_x+n,\ x\,S_a^2+(1+a+x)\,D_x+(n-x)\},\ \{-t,-t,-t\}\Big\}\Big\}$$

Bivariate rational integrands

```
f = 1 / (1 - y - x / y - x (1 - y^2))
```

$$\frac{1}{1-\frac{x}{y}-y-x\left(1-y^2\right)}$$

Our routine Test does not work directly on these examples, but Christoph Koutschan suggested changing the order of the derivations below, which solves the problem:

```
ann = Annihilator[f, {Der[y], Der[x]}]
```

$$\left\{\left(-x+y-x\,y-y^2+x\,y^3\right)D_x+\left(-1-y+y^3\right),\ \left(-x\,y+y^2-x\,y^2-y^3+x\,y^4\right)D_y+\left(x-y^2+2x\,y^3\right)\right\}$$

```
CreativeTelescoping[ann, Der[y], {Der[x]}]
```

$$\{\{(-1+15x-96x^2+335x^3-592x^4+335x^5+345x^6)\,D_x^2+(10-88x+370x^2-852x^3+550x^4+1380x^5)\,D_x+(-12+30x+72x^2-120x^3+690x^4)\},$$
$$\{(x^2-6x^3+48x^4-155x^5-2xy+14x^2y-108x^3y+406x^4y-310x^5y+y^2-7xy^2+59x^2y^2-219x^3y^2+213x^4y^2-285x^5y^2-2y^3+16xy^3-118x^2y^3+444x^3y^3-518x^4y^3+400x^5y^3+y^4-5xy^4+26x^2y^4-10x^3y^4-368x^4y^4+395x^5y^4-2xy^5+20x^2y^5-116x^3y^5+330x^4y^5-260x^5y^5)\,/\,(x^2y(-x+y-xy-y^2+xy^3))\}\}$$

```
Do[f = 1 / (1 - y - x / y - x * (1 - y ^ d));
 ann = Annihilator[f, {Der[y], Der[x]}];
 tt = Timing[CreativeTelescoping[ann, Der[y], {Der[x]}]];
 Print[d, "  ", tt[[1]]], {d, 12}]
```

```
1  0.036701
2  0.07005
3  0.23705
4  0.551711
5  1.23411
6  2.48882
7  4.81169
8  8.1914
9  13.7818
10  22.2077
11  35.2914
12  54.9072
```

Bivariate Hyperexponential Integrals

```
f = (62 * x - 85 * y - 26) * Sqrt[(-99 * x + 33 * y + 62) / (-66 * x - 63 * y + 29)] *
  Exp[(-51 * x + 86 * y - 54) / (-51 * x - 12 * y + 68)] / (-44 * x - y + 87) ^ 5
```

$$\left(e^{\frac{-54-51x+86y}{68-51x-12y}}\,(-26+62x-85y)\sqrt{\frac{62-99x+33y}{29-66x-63y}}\right)\Big/\,(87-44x-y)^5$$

Here, exchanging the order of the derivations does not help, so we use FindCreativeTelescoping instead of CreativeTelescoping:

```
Timing[FindCreativeTelescoping[f, Der[y], {Der[x]}]][[1]]
```

```
20.1508
```

```
? FindCreativeTelescoping
```

FindCreativeTelescoping[f, {d1, ..., dj}, {op1, ..., opk}] or FindCreativeTelescoping[ann, {d1, ..., dj}] finds creative telescoping relations for the given function f (resp. in the given ∂–finite ideal ann annihilating some function f). In particular it returns {{p1, ..., pm}, {{q11, ..., q1j}, ..., {qm1, ..., qmj}}}, wherein all entries are OrePolynomials such that pi + d1*qi1 + ... + dj*qij is in the annihilator of f for all 1 <= i <= m. FindCreativeTelescoping makes an ansatz with explicit denominators, in contrast to Chyzak's algorithm (see the command CreativeTelescoping) where the denominators have to be computed later. Modular computations are used to get the denominators right.

The following options can be given:

Degree –> n: to limit the degrees of the summation/integration variables in the numerators by n (each variable separately).

Support –> {...}: to give the support of the telescoper.

Mode –> Automatic: finds the right ansatz and does the non–modular computation automatically.

Mode –> FindSupport: does only the modular computations and returns a list of options specifying the ansatz which can again be given to FindCreativeTelescoping again.

Mode –> Modular: takes a fixed ansatz and computes homomorphic images (to be specified by the options OrePolynomialSubstitute and Modulus, and FileNames has to give a StringForm where on the disk to store the results).

Denominator –> d: to give a common denominator that is tried for finding a creative telescoping relation.

"MinimizeDenominators" –> True: whether to minimize the denominators in the delta parts using homomorphic images.

A bigger example:

```
f = (62 * x - 85 * y - 26) * Sqrt[(-99 * x + 33 * y + 62) / (-66 * x - 63 * y + 29)] *
  Exp[(-99 * x ^ 2 + 62 * x * y + 23 * y ^ 2 + 40 * x + 41 * y - 87) /
     (-53 * x ^ 2 + 9 * x * y + 44 * y ^ 2 - 89 * x + 40 * y + 98)] / (-44 * x - y + 87) ^ 5
```

$$\left(e^{\frac{-87+40x-99x^2+41y+62xy+23y^2}{98-89x-53x^2+40y+9xy+44y^2}}(-26+62x-85y)\sqrt{\frac{62-99x+33y}{29-66x-63y}}\right)\Big/(87-44x-y)^5$$

```
TimeConstrained[
 Timing[FindCreativeTelescoping[f, Der[y], {Der[x]}]], TimeBound]
```

$$\left\{2206.76, \left\{\left\{(\cdots 1 \cdots)\, D_x^5 + (\cdots 1 \cdots)\, D_x^4 + (\cdots 1 \cdots)\, D_x^3 + (\cdots 1 \cdots)\, D_x^2 + (\cdots 1 \cdots)\, D_x + (\cdots 1 \cdots)\right\}, \left\{\left\{\frac{\cdots 1 \cdots}{\cdots 1 \cdots}\right\}\right\}\right\}\right\}$$

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Bivariate Mixed Terms

```
f = (1 + x / (n^2 + 1)) * ((x + 1) ^2 / (x - 4) / (x - 3) ^2 / (x^2 - 5) ^3) ^n *
  Sqrt[x^2 - 5] * Exp[(x^3 + 1) / x / (x - 3) / (x - 4) ^2]
```

$$e^{\frac{1+x^3}{(-4+x)^2\,(-3+x)\,x}}\left(1+\frac{x}{1+n^2}\right)\left(\frac{(1+x)^2}{(-4+x)\,(-3+x)^2\,\left(-5+x^2\right)^3}\right)^n\sqrt{-5+x^2}$$

```
Test[f, Der[x], {S[n]}]
```

$$\{231.954,\ \{\{(\cdots 1 \cdots)\,S_n^9+(\cdots 1 \cdots)\,S_n^8+(\cdots 1 \cdots)\,S_n^7+(\cdots 1 \cdots)\,S_n^6+\cdots 2 \cdots+\cdots 1 \cdots+(\cdots 1 \cdots)\,S_n^2+(\cdots 1 \cdots)\,S_n+(\cdots 1 \cdots)\},\ \cdots 1 \cdots\}\}$$

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```
Test[((z^2 - 1) / 2 / (z - x)) ^n * (1 - z) ^alpha * (1 + z) ^beta / (z - x), Der[z], {S[n]}]
```

$$\{0.217684,$$

$$\{\{(16+16\,\text{alpha}+4\,\text{alpha}^2+16\,\text{beta}+8\,\text{alpha}\,\text{beta}+4\,\text{beta}^2+32\,n+20\,\text{alpha}\,n+2\,\text{alpha}^2\,n+20\,\text{beta}\,n+4\,\text{alpha}\,\text{beta}\,n+2\,\text{beta}^2\,n+20\,n^2+6\,\text{alpha}\,n^2+6\,\text{beta}\,n^2+4\,n^3)\,S_n^2+$$

$$(-3\,\text{alpha}^2-\text{alpha}^3-\text{alpha}^2\,\text{beta}+3\,\text{beta}^2+\text{alpha}\,\text{beta}^2+\text{beta}^3-2\,\text{alpha}^2\,n+2\,\text{beta}^2\,n-24\,x-26\,\text{alpha}\,x-9\,\text{alpha}^2\,x-\text{alpha}^3\,x-26\,\text{beta}\,x-18\,\text{alpha}\,\text{beta}\,x-3\,\text{alpha}^2\,\text{beta}\,x-9\,\text{beta}^2\,x-3\,\text{alpha}\,\text{beta}^2\,x-\text{beta}^3\,x-52\,n\,x-36\,\text{alpha}\,n\,x-6\,\text{alpha}^2\,n\,x-36\,\text{beta}\,n\,x-12\,\text{alpha}\,\text{beta}\,n\,x-6\,\text{beta}^2\,n\,x-36\,n^2\,x-12\,\text{alpha}\,n^2\,x-12\,\text{beta}\,n^2\,x-8\,n^3\,x)\,S_n+$$

$$(8+10\,\text{alpha}+2\,\text{alpha}^2+10\,\text{beta}+12\,\text{alpha}\,\text{beta}+2\,\text{alpha}^2\,\text{beta}+2\,\text{beta}^2+2\,\text{alpha}\,\text{beta}^2+20\,n+16\,\text{alpha}\,n+2\,\text{alpha}^2\,n+16\,\text{beta}\,n+8\,\text{alpha}\,\text{beta}\,n+2\,\text{beta}^2\,n+16\,n^2+6\,\text{alpha}\,n^2+6\,\text{beta}\,n^2+4\,n^3)\},$$

$$\{\frac{1}{2\,(x-z)}(-4-4\,\text{alpha}-\text{alpha}^2-4\,\text{beta}-2\,\text{alpha}\,\text{beta}-\text{beta}^2-6\,n-3\,\text{alpha}\,n-3\,\text{beta}\,n-2\,n^2-4\,\text{alpha}\,x-\text{alpha}^2\,x+4\,\text{beta}\,x+\text{beta}^2\,x-2\,\text{alpha}\,n\,x+2\,\text{beta}\,n\,x+4\,\text{alpha}\,z+\text{alpha}^2\,z-4\,\text{beta}\,z-\text{beta}^2\,z+2\,\text{alpha}\,n\,z-2\,\text{beta}\,n\,z+8\,x\,z+6\,\text{alpha}\,x\,z+\text{alpha}^2\,x\,z+6\,\text{beta}\,x\,z+2\,\text{alpha}\,\text{beta}\,x\,z+\text{beta}^2\,x\,z+12\,n\,x\,z+4\,\text{alpha}\,n\,x\,z+4\,\text{beta}\,n\,x\,z+4\,n^2\,x\,z+2\,\text{alpha}\,z^2+\text{alpha}^2\,z^2+2\,\text{beta}\,z^2+2\,\text{alpha}\,\text{beta}\,z^2+\text{beta}^2\,z^2+2\,\text{alpha}\,n\,z^2+2\,\text{beta}\,n\,z^2+4\,\text{alpha}\,x\,z^2+\text{alpha}^2\,x\,z^2-4\,\text{beta}\,x\,z^2-\text{beta}^2\,x\,z^2+2\,\text{alpha}\,n\,x\,z^2-2\,\text{beta}\,n\,x\,z^2-4\,\text{alpha}\,z^3-\text{alpha}^2\,z^3+4\,\text{beta}\,z^3+\text{beta}^2\,z^3-2\,\text{alpha}\,n\,z^3+2\,\text{beta}\,n\,z^3-8\,x\,z^3-6\,\text{alpha}\,x\,z^3-\text{alpha}^2\,x\,z^3-6\,\text{beta}\,x\,z^3-2\,\text{alpha}\,\text{beta}\,x\,z^3-\text{beta}^2\,x\,z^3-12\,n\,x\,z^3-4\,\text{alpha}\,n\,x\,z^3-4\,\text{beta}\,n\,x\,z^3-4\,n^2\,x\,z^3+4\,z^4+2\,\text{alpha}\,z^4+2\,\text{beta}\,z^4+6\,n\,z^4+\text{alpha}\,n\,z^4+\text{beta}\,n\,z^4+2\,n^2\,z^4)\}\}\}$$

```
Test[(1 + x) ^ (100 * n) / x ^ (n + 1), Der[x], {S[n]}]
```

{89.5272, {{(⋯ 1 ⋯) S_n + (⋯ 1 ⋯)}, {⋯ 1 ⋯}}}

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```
f = 4 * (x * (12 * x + 10) ^ 2 / (87 * x + 37)) ^ (-n - 1) *
  x * (6 * x + 5) * (1044 * x ^ 2 + 666 * x + 185) / (87 * x + 37) ^ 2
```

$$\left(4\,x\,(5+6\,x)\,\left(\frac{x\,(10+12\,x)^2}{37+87\,x}\right)^{-1-n}\left(185+666\,x+1044\,x^2\right)\right)\Big/\,(37+87\,x)^2$$

```
Test[f, Der[x], {S[n]}]
```

$$\{0.103724,\ \{\{(28\,968\,000+244\,240\,000\,n+255\,174\,000\,n^2+70\,148\,000\,n^3)\,S_n^2+ (521\,424+98\,781\,996\,n+222\,482\,304\,n^2+104\,077\,896\,n^3)\,S_n+ (-61\,679\,781\,n+69\,142\,815\,n^2+108\,433\,494\,n^3)\},$$

$$\{(91\,707\,256\,500+773\,218\,045\,000\,n+807\,833\,038\,875\,n^2+222\,075\,415\,250\,n^3+ 1\,458\,956\,818\,875\,x+8\,163\,839\,566\,200\,n\,x+8\,481\,824\,965\,275\,n^2\,x+ 2\,457\,048\,480\,900\,n^3\,x+7\,570\,839\,656\,925\,x^2+35\,954\,556\,193\,260\,n\,x^2+ 39\,151\,775\,566\,800\,n^2\,x^2+12\,516\,065\,579\,160\,n^3\,x^2+18\,649\,288\,387\,656\,x^3+ 87\,366\,431\,049\,066\,n\,x^3+103\,874\,052\,113\,856\,n^2\,x^3+37\,075\,911\,988\,986\,n^3\,x^3+ 24\,306\,010\,315\,344\,x^4+129\,682\,851\,318\,018\,n\,x^4+172\,427\,616\,936\,846\,n^2\,x^4+ 67\,813\,119\,407\,592\,n^3\,x^4+16\,295\,493\,624\,192\,x^5+118\,502\,478\,507\,744\,n\,x^5+ 177\,570\,940\,399\,128\,n^2\,x^5+75\,363\,955\,515\,576\,n^3\,x^5+4\,316\,597\,793\,504\,x^6+ 60\,389\,698\,114\,656\,n\,x^6+102\,026\,347\,610\,400\,n^2\,x^6+45\,953\,247\,289\,248\,n^3\,x^6+ 13\,909\,037\,334\,624\,n\,x^7+26\,135\,130\,650\,112\,n^2\,x^7+12\,226\,093\,315\,488\,n^3\,x^7)\Big/ \left((1+n)\,x\,(5+6\,x)^3\,(185+666\,x+1044\,x^2)\right)\}\}\}$$

```
f = 4 * (x * (76 * x ^ 2 - 90 * x + 92) ^ 2 / (82 * x ^ 2 + 58 * x + 65)) ^ (-n - 1) *
  x * (38 * x ^ 2 - 45 * x + 46) *
  (9348 * x ^ 4 + 5126 * x ^ 3 + 3358 * x ^ 2 - 8775 * x + 2990) / (82 * x ^ 2 + 58 * x + 65) ^ 2
```

$$\frac{1}{\left(65+58\,x+82\,x^2\right)^2}\,4\,x\,\left(46-45\,x+38\,x^2\right) \left(\frac{x\,\left(92-90\,x+76\,x^2\right)^2}{65+58\,x+82\,x^2}\right)^{-1-n}\left(2990-8775\,x+3358\,x^2+5126\,x^3+9348\,x^4\right)$$

```
Test[f, Der[x], {S[n]}][[1]]
```

1.37184

```
f = (x * (33 * x^4 - 59 * x^3 - 9 * x^2 + 22 * x - 99)^2 /
      (86 * x^4 - 37 * x^3 + 94 * x^2 + 80 * x - 31))^
    (-n - 1) * x * (33 * x^4 - 59 * x^3 - 9 * x^2 + 22 * x - 99) *
   (14190 * x^8 - 22548 * x^7 + 29672 * x^6 - 7836 * x^5 - 14523 * x^4 + 4665 * x^3 +
      14221 * x^2 - 2046 * x + 3069) / (86 * x^4 - 37 * x^3 + 94 * x^2 + 80 * x - 31)^2
```

$$\left(x\left(-99+22x-9x^2-59x^3+33x^4\right)\left(\left(x\left(-99+22x-9x^2-59x^3+33x^4\right)^2\right)\Big/\left(-31+80x+94x^2-37x^3+86x^4\right)\right)^{-1-n}\left(3069-2046x+14221x^2+4665x^3-14523x^4-7836x^5+29672x^6-22548x^7+14190x^8\right)\right)\Big/\left(-31+80x+94x^2-37x^3+86x^4\right)^2$$

```
Test[f, Der[x], {S[n]}][[1]]
```

128.008

Integrals with Gegenbauer polynomials

```
f = x^l * GegenbauerC[m, mu, x] * GegenbauerC[n, nu, x] * (-x^2 + 1)^(nu - 1/2)
```

$x^l\left(1-x^2\right)^{-\frac{1}{2}+nu}$ GegenbauerC[m, mu, x] GegenbauerC[n, nu, x]

```
Test[f /. l → 0, Der[x], {S[m], S[n], S[mu], S[nu]}]
```

$$\{516.366,$$
$$\{\{(-4\,nu + m^2\,nu + 4\,mu\,nu + 2\,m\,mu\,nu - 4\,n\,nu + 2\,mu\,n\,nu - n^2\,nu - 8\,nu^2 + 4\,mu\,nu^2 - 4\,n\,nu^2 - 4\,nu^3)\,S_{nu} + (n - mu\,n + n^2 - mu\,n^2 + 2\,nu - 2\,mu\,nu + 5\,n\,nu - 4\,mu\,n\,nu + n^2\,nu + 6\,nu^2 - 4\,mu\,nu^2 + 4\,n\,nu^2 + 4\,nu^3),$$
$$(4\,mu^2 - 4\,mu\,nu)\,S_{mu} + (-m^2 - 4\,m\,mu - 4\,mu^2 + n^2 + 2\,m\,nu + 4\,mu\,nu + 2\,n\,nu),$$
$$(n + m\,n - n^2 + 2\,nu + 2\,m\,nu - 2\,n\,nu)\,S_m + (1 - m - 2\,mu + 2\,n - m\,n - 2\,mu\,n + n^2 + 2\,nu + 2\,n\,nu)\,S_n,$$
$$(-8 + 2\,m^2 + 8\,mu + 4\,m\,mu - 20\,n + 3\,m^2\,n + 16\,mu\,n + 6\,m\,mu\,n - 18\,n^2 + m^2\,n^2 + 10\,mu\,n^2 + 2\,m\,mu\,n^2 - 7\,n^3 + 2\,mu\,n^3 - n^4 - 16\,nu + 8\,mu\,nu - 32\,n\,nu + 12\,mu\,n\,nu - 20\,n^2\,nu + 4\,mu\,n^2\,nu - 4\,n^3\,nu - 8\,nu^2 - 12\,n\,nu^2 - 4\,n^2\,nu^2)\,S_n^2 + (-m^2\,n - 2\,m\,mu\,n - m^2\,n^2 + 2\,mu\,n^2 - 2\,m\,mu\,n^2 + n^3 + 2\,mu\,n^3 + n^4 - 2\,m^2\,nu - 4\,m\,mu\,nu - 4\,m^2\,n\,nu + 4\,mu\,n\,nu - 8\,m\,mu\,n\,nu + 2\,n^2\,nu + 8\,mu\,n^2\,nu + 4\,n^3\,nu - 4\,m^2\,nu^2 - 8\,m\,mu\,nu^2 + 8\,mu\,n\,nu^2 + 4\,n^2\,nu^2)\},$$
$$\{-\frac{2\,mu\,nu\,(-1 + x)\,(1 + x)}{x}\,S_{mu}\,S_{nu} + \frac{1}{x}nu\,(-1 - m - 2\,mu - n - 2\,nu + 2\,x^2 + m\,x^2 + n\,x^2 + 2\,nu\,x^2)\,S_{nu} + \frac{1}{2\,x}(n + n^2 + 2\,nu + 4\,n\,nu + 4\,nu^2 - n\,x^2 - n^2\,x^2 - 2\,nu\,x^2 - 4\,n\,nu\,x^2 - 4\,nu^2\,x^2),$$
$$\frac{2\,mu\,(-1 + x^2)}{x}\,S_{mu} + \frac{2\,nu}{x}\,S_{nu} + \frac{1}{x}(m + 2\,mu - n - 2\,nu - m\,x^2 - 2\,mu\,x^2 + n\,x^2 + 2\,nu\,x^2),$$
$$-\frac{4\,mu\,nu\,(-1 + x)\,(1 + x)}{(1 + m)\,x^2}\,S_{mu}\,S_{nu} - ((2\,mu\,(n + 2\,nu - 2\,n\,x^2 - 4\,nu\,x^2 + n\,x^4 + 2\,nu\,x^4)) / ((1 + m)\,x^2))\,S_{mu} + \frac{2\,nu\,(-m - 2\,mu + x^2 + m\,x^2)}{(1 + m)\,x^2}\,S_{nu} + (m\,n + 2\,mu\,n + 2\,m\,nu + 4\,mu\,nu - m\,n\,x^2 - 2\,mu\,n\,x^2 - 2\,m\,nu\,x^2 - 4\,mu\,nu\,x^2) / ((1 + m)\,x^2),$$
$$\frac{8\,mu\,nu\,(1 + n + nu)\,(-1 + x)\,(1 + x)}{x}\,S_{mu}\,S_{nu} - \frac{1}{x}4\,nu\,(1 + n + nu)\,(-1 - m - 2\,mu - n - 2\,nu + 2\,x^2 + m\,x^2 + n\,x^2 + 2\,nu\,x^2)\,S_{nu} + \frac{1}{x}2\,(1 + n + nu)\,(-n - n^2 - 2\,nu - 4\,n\,nu - 4\,nu^2 + n\,x^2 + n^2\,x^2 + 2\,nu\,x^2 + 4\,n\,nu\,x^2 + 4\,nu^2\,x^2)\}\}\}$$

```
Test[f, Der[x], {S[m], S[n], S[l], S[mu], S[nu]}]
```

$Aborted

```
F = (x + a) ^ (g + lambda - 1) * (a - x) ^ (beta - 1) *
  GegenbauerC[m, g, x / a] * GegenbauerC[n, lambda, x / a]
```

$$(a - x)^{-1+beta}\,(a + x)^{-1+g+lambda}\,GegenbauerC\left[m,\,g,\,\frac{x}{a}\right]\,GegenbauerC\left[n,\,lambda,\,\frac{x}{a}\right]$$

```
Test[f, Der[x], {S[m], S[n], Der[a], S[beta]}]
```

$Aborted